International Journal of Advanced and Applied Sciences, 2(10) 2015, Pages: 32-40

Contents lists available at Science-Gate

International Journal of Advanced and Applied Sciences

Journal homepage: http://www.science-gate.com/IJAAS.html

Nonlinear vibration analysis of piezoelectric nano-composite pipes using DQM

Rasool Abedzadeh, Reza Kolahchi*, Mohammad Mehdi Heydari

Department of Civil Engineering, Khomein Branch, Islamic Azad University, Khomein, Iran

ARTICLE INFO

Article history:
Received 27 October 2015
Received in revised form
20 December 2015
Accepted 25 December 2015

Keywords:
Nonlinear vibration
DWBNNTs
Cylindrical shell
Pasternak medium
DQM

ABSTRACT

In the present study, nonlinear vibration of piezoelectric nano-composite pipes resting on elastic foundation is investigated based on classical cylindrical shell theory. The pipe is reinforced with double walled boron nitride nanotubes (DWBNNTs). The surrounding elastic foundation is modeled with Pasternak medium. Micromechanical model is applied in order to obtain the characteristics of the equivalent composite. Using energy method, the motion equations are derived based on Hamilton's principal. The nonlinear frequency of system is obtained utilizing the differential quadrature method (DQM). The effects of different parameters such as orientation angle and percentage of DWBNNTs, geometrical parameters of shell and elastic foundation on the vibration of pipe are investigated. Results showed that with increasing the volume present of DWBNNTs in pipe, the frequency of structure increases.

1. Introduction

Circular cylindrical shells are used in a great variety of engineering applications in mechanical, and process industries, ranging from storage and transport of high-pressure gases and liquids, too much smaller Nano-scale applications in smart structures such as sensors and actuators and hence are required to be modeled mathematically. A good understanding of their mechanical behavior, including vibration, bending and wave propagation response, is therefore required for successful design practices. Improving mechanical behaviors (e.g. increasing stability and reduction of weight) of such structures in composite applications have also received considerable attention amongst researchers in the last two decades. Most studies to-date is limited to linear vibration, despite the fact that deformations of cylindrical shell are nonlinear in nature. Having considered the geometrical nonlinearities, more precise dynamic properties of cylindrical shell could be obtained to extend the engineering applications, especially in Nano-composites.

Mechanical analysis of cylindrical shell has been done by many researchers. Effects of internal flow on the vibration of a cylindrical shell were investigated by Païdoussis and Denise (1972), Amabili and Garziera (2002) and Païdoussis (1998). None of these studies considered smart structures such as PVDF, new polymeric piezoelectric materials offering advantages including flexibility in thermoplastic conversion techniques, excellent dimensional stability, abrasion and corrosion resistance, high strength, ability to maintain the superior mechanical properties at elevated temperature (Topolov and Bowen, 2008). Arani et al. (2011 and 2012), carried out a stress analysis in cylinder and spheres made from piezoelectric materials using analytical method and ANSYS software. In another study, the embedding of piezoelectric materials in the form of fibers into a polymer matrix was implemented by Bent et al. (1995). Free vibration of composite plates and cylindrical shell panels were studied by Messina and Soldatos (1999) using a higher-order theory. Free vibration and buckling analysis of composite cylindrical shells conveying hot fluid was proposed by Kadoli and Ganesan (2003). Vibration and buckling of cross-ply laminated composite circular cylindrical shells were studied by Matsuna (2007) based on a global higher-order theory. Post buckling instability of nonlinear Nano-Beam with geometric imperfection embedded in elastic foundation is studied by Mohammadi et al. (2014).

With respect to developmental works on analysis of the cylindrical shells, it should be noted that none of the research mentioned above, have considered smart composites and their specific characteristics. Micromechanical modeling which has the potential to take into account the electrical load was used by Tan and Tong (2001) for studying an imperfect textile composite. However, neither the matrix nor the reinforced material used in the composite employed in this work was smart. Rahmani et al. (2010) investigated free vibration response of

* Corresponding Author.
Email Address: r.kolahchi@gmail.com

composite sandwich cylindrical shells with flexible core. Buckling and vibration analysis of plate/shell structures via a smoothed quadrilateral flat shell element with in-plane rotations were studied by Nguyen-Van (2011). Electro-thermo-mechanical nonlinear buckling of a piezoelectric polymeric cylindrical shell reinforced by DWBNNTs was studied by Mosallaie Barzoki et al. (2013).

None of the above mentioned works dose not studied nonlinear vibration analysis of piezoelectric pipes. However, in the present work, nonlinear vibration of piezoelectric pipes reinforced with DWBNNTs embedded in a Pasternak medium is studied. DQM is utilized for obtaining the nonlinear frequency of system. The influence of geometrical parameters of shell, elastic foundation, orientation angle and percentage of DWBNNTs in polymer on the vibration of pipe is investigated.

2. Mathematical modeling

2.1. Constitutive equations of piezoelectric materials

In a piezoelectric material, application of an electric field to it will cause a strain proportional to the mechanical field strength, and vice versa. The constitutive equation for stresses σ and strains Ɛ matrix on the mechanical side, as well as flux density D and field strength E matrix on the electrostatic side, may be arbitrarily combined as follows (Arani et al. 2015):

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{\theta\theta} \\ \sigma_{zz} \\ \sigma_{\theta z} \\ \sigma_{zx} \\ \sigma_{x\theta} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \times \left(\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{\theta\theta} \\ \varepsilon_{zz} \\ 2\varepsilon_{\theta z} \\ 2\varepsilon_{zx} \\ 2\varepsilon_{x\theta} \end{Bmatrix} \right) - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ e_{14} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} E_x \\ E_\theta \\ E_z \end{Bmatrix}, \qquad (1)$$

$$\begin{Bmatrix} D_x \\ D_\theta \\ D_z \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & e_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \left(\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{\theta\theta} \\ \varepsilon_{zz} \\ 2\varepsilon_{\theta z} \\ 2\varepsilon_{zx} \\ 2\varepsilon_{x\theta} \end{Bmatrix} \right) + \begin{bmatrix} \in_{11} & 0 & 0 \\ 0 & \in_{22} & 0 \\ 0 & 0 & \in_{33} \end{bmatrix} \begin{Bmatrix} E_x \\ E_\theta \\ E_z \end{Bmatrix}, \qquad (2)$$

Where $C_{ij}, e_{ij}, \mathcal{E}_{ii}, (i,j = 1, \dots 6)$ are elastic constants, piezoelectric constants and dielectric constants respectively? Also, the electric field may be written in term of electric potential as:

$$E_k = -\nabla \phi. \qquad (3)$$

The transformed elastic constants are defined as:

$$[Q] = [R][C][R]^T, \qquad (4)$$

Where [R] is the transfer matrix defined as:

$$[R] = \begin{bmatrix} \cos^2(\theta) & \sin^2(\theta) & 0 & 0 & 0 & -\sin(2\theta) \\ \sin^2(\theta) & \cos^2(\theta) & 0 & 0 & 0 & \sin(2\theta) \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\theta) & \sin(\theta) & 0 \\ 0 & 0 & 0 & -\sin(\theta) & \cos(\theta) & 0 \\ \sin(\theta)\cos(\theta) & -\sin(\theta)\cos(\theta) & 0 & 0 & 0 & \cos^2(\theta) - \sin^2(\theta) \end{bmatrix}, \qquad (5)$$

Here, Θ is the angle between the global and local cylindrical co-ordinates, which corresponds to the orientation angle between DWBNNTs and the main axis of the matrix. Based on classical shell theory, the constitute equations of (1) and (2) may be simplified as:

$$\begin{Bmatrix} \sigma_x \\ \sigma_\theta \\ \tau_{x\theta} \\ D_x \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & -e_{11} \\ Q_{12} & Q_{22} & 0 & -e_{12} \\ 0 & 0 & Q_{66} & 0 \\ e_{11} & e_{12} & 0 & \in_{11} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ \gamma_{x\theta} \\ E_x \end{Bmatrix}, \qquad (6)$$

Based on micro-mechanical model, the mechanical and electrical properties of the composite are (Mosallaie Barzoki et al. 2013):

$$Q_{11} = \frac{C_{11}^r C_{11}^m}{\rho C_{11}^m + (1-\rho) C_{11}^r}, \qquad (7)$$

$$Q_{12} = C_{11} \left[\frac{\rho C_{12}^r}{C_{11}^r} + \frac{+(1-\rho) C_{12}^m}{C_{11}^m} \right], \qquad (8)$$

$$Q_{22}=\rho C_{22}^{r}+(1-\rho)C_{22}^{m}+\frac{C_{12}^{2}}{C_{11}}-\frac{\rho(C_{12}^{r})^{2}}{C_{11}^{r}}-\frac{(1-\rho)(C_{12}^{m})^{2}}{C_{11}^{m}}, \tag{9}$$

$$Q_{66}=\frac{C_{66}^{r}C_{66}^{m}}{\rho C_{66}^{m}+(1-\rho)C_{66}^{r}}, \tag{10}$$

$$e_{11}=C_{11}\left[\frac{\rho e_{11}^{r}}{C_{11}^{r}}+\frac{+(1-\rho)e_{11}^{m}}{C_{11}^{m}}\right], \tag{11}$$

$$e_{12}=\rho e_{12}^{r}+(1-\rho)e_{12}^{m}+\frac{C_{12}e_{11}}{C_{11}}-\frac{\rho C_{12}^{r}e_{11}^{r}}{C_{11}^{r}}-\frac{(1-\rho)C_{12}^{m}e_{11}^{m}}{C_{11}^{m}}, \tag{12}$$

$$\in_{11}=\frac{C}{B^{2}+AC}, \tag{13}$$

Where:

$$A=\frac{\rho C_{55}^{r}}{(e_{15}^{r})^{2}+C_{55}^{r}\in_{11}^{r}}+\frac{(1-\rho)C_{55}^{m}}{(e_{15}^{m})^{2}+C_{55}^{m}\in_{11}^{m}}, \tag{14}$$

$$B=\frac{\rho e_{15}^{r}}{(e_{15}^{r})^{2}+C_{55}^{r}\in_{11}^{r}}+\frac{(1-\rho)e_{15}^{m}}{(e_{15}^{m})^{2}+C_{55}^{m}\in_{11}^{m}}, \tag{15}$$

$$C=\frac{\rho\in_{11}^{r}}{(e_{15}^{r})^{2}+C_{55}^{r}\in_{11}^{r}}+\frac{(1-\rho)\in_{11}^{m}}{(e_{15}^{m})^{2}+C_{55}^{m}\in_{11}^{m}}, \tag{16}$$

Superscripts r and m refer to the reinforced and matrix components of the composite, respectively ρ is also the volume percent of the reinforced DWBNNTs in matrix.

2.2. Strain-displacement relation

Based on classical shell model, the displacement components are written as (Brush and Almroth, 1975):

$$\begin{aligned}U(x,\theta,z,t)&=u(x,\theta,t)-z\frac{\partial w(x,\theta,t)}{\partial x},\\ V(x,\theta,z,t)&=v(x,\theta,t)-z\frac{\partial w(x,\theta,t)}{R\partial\theta},\\ W(x,\theta,z,t)&=w(x,\theta,t),\end{aligned} \tag{17}$$

where, U,V,W are the displacements of an arbitrary point of the shell in the axial, circumferential and radial directions, respectively, u, v, w are the displacements of points on the middle surface of the shell and z is the distance of the arbitrary point of the shell from the middle surface. Hence, the mechanical strain components ε_{xx}, $\varepsilon_{\theta\theta}$, $\varepsilon_{x\theta}$ at an arbitrary point of the shell are related to the middle surface strains $\varepsilon_{x,0}$ $\varepsilon_{\theta,0}$, $\varepsilon_{x\theta,0}$ and changes in the curvature and torsion of the middle surface k_x, k_θ, $k_{x\theta}$ as follows:

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_\theta\\ \gamma_{x\theta}\end{Bmatrix}_{shell}=\begin{Bmatrix}\varepsilon_{xm}\\ \varepsilon_{\theta m}\\ \gamma_{x\theta m}\end{Bmatrix}_{L}+\begin{Bmatrix}\varepsilon_{xm}\\ \varepsilon_{\theta m}\\ \gamma_{x\theta m}\end{Bmatrix}_{NL}-z\begin{Bmatrix}k_x\\ k_\theta\\ k_{x\theta}\end{Bmatrix}=\begin{pmatrix}\dfrac{\partial u}{\partial x}\\ \dfrac{\partial v}{R\partial\theta}+\dfrac{w}{R}\\ \dfrac{\partial u}{R\partial\theta}+\dfrac{\partial v}{\partial x}\end{pmatrix}+\begin{pmatrix}\dfrac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^{2}\\ \dfrac{1}{2}\left(\dfrac{\partial w}{R\partial\theta}\right)^{2}\\ \dfrac{\partial w}{\partial x}\dfrac{\partial w}{R\partial\theta}\end{pmatrix}-z\begin{pmatrix}\dfrac{\partial^{2}w}{\partial x^{2}}\\ \dfrac{\partial^{2}w}{R^{2}\partial\theta^{2}}\\ 2\dfrac{\partial^{2}w}{R\partial x\partial\theta}\end{pmatrix}. \tag{18}$$

2.3. Energy method

The total potential energy of the pipe is the sum of strain energy, U, kinetic energy K, and the work W done by the applied load. The strain energy is:

$$U_s=\int_V\left(\sigma_x\varepsilon_x+\sigma_\theta\varepsilon_\theta+\sigma_{x\theta}\gamma_{x\theta}-D_xE_x\right)dV, \tag{19}$$

Strain energy by combining Eq. (18) and Eq. (19), may be written as:

$$\begin{aligned}U_s=\int_A\Bigg(&N_x\left(\frac{\partial u}{\partial x}+0.5\left(\frac{\partial w}{\partial x}\right)^{2}\right)-M_x\frac{\partial^{2}w}{\partial x^{2}}+N_\theta\left(\frac{\partial v}{R\partial\theta}+\frac{w}{R}+0.5\left(\frac{\partial w}{R\partial\theta}\right)^{2}\right)\\ &-M_\theta\frac{\partial^{2}w}{R^{2}\partial\theta^{2}}+N_{x\theta}\left(\frac{\partial u}{R\partial\theta}+\frac{\partial v}{\partial x}+\frac{\partial w}{R\partial\theta}\frac{\partial w}{\partial x}\right)-2M_{x\theta}\frac{\partial^{2}w}{R\partial\theta\partial x}+G_x\frac{\partial\phi}{\partial x}\Bigg)dA\end{aligned} \tag{20}$$

Where the internal forces and moments may be expressed as:

$$\begin{Bmatrix}N_x\\ N_\theta\\ N_{x\theta}\\ G_x\end{Bmatrix}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\begin{Bmatrix}\sigma_x\\ \sigma_\theta\\ \tau_{x\theta}\\ D_x\end{Bmatrix}dz\ , \tag{21}$$

$$\begin{Bmatrix}M_x\\ M_\theta\\ M_{x\theta}\\ Q_x\end{Bmatrix}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\begin{Bmatrix}\sigma_x\\ \sigma_\theta\\ \tau_{x\theta}\\ D_x\end{Bmatrix}zdz\ . \tag{22}$$

Substituting equation 1 into equation 21 and equation 22 yields:

$$N_x=\left(hC_{11}\left(\frac{\partial u}{\partial x}+0.5\left(\frac{\partial w}{\partial x}\right)^2-\alpha_x\Delta T\right)+C_{12}\left(\frac{\partial v}{R\partial\theta}+\frac{w}{R}+0.5\left(\frac{\partial w}{R^2\partial\theta}\right)^2-\alpha_\theta\Delta T\right)+e_{11}\frac{\partial\phi}{\partial x}\right),$$

$$N_\theta=\left(hC_{12}\left(\frac{\partial u}{\partial x}+0.5\left(\frac{\partial w}{\partial x}\right)^2-\alpha_x\Delta T\right)+C_{22}\left(\frac{\partial v}{R\partial\theta}+\frac{w}{R}+0.5\left(\frac{\partial w}{R^2\partial\theta}\right)^2-\alpha_\theta\Delta T\right)+e_{12}\frac{\partial\phi}{\partial x}\right),\qquad(23)$$

$$N_{x\theta}=h\left(C_{66}\left(\frac{\partial u}{R\partial\theta}+\frac{\partial v}{\partial x}+\frac{\partial w}{R\partial\theta}\frac{\partial w}{\partial x}\right)\right),$$

$$G_x=e_{11}\left(\frac{\partial u}{\partial x}+0.5\left(\frac{\partial w}{\partial x}\right)^2-\alpha_x\Delta T\right)+e_{12}\left(\frac{\partial v}{R\partial\theta}+\frac{w}{R}+0.5\left(\frac{\partial w}{R\partial\theta}\right)^2-\alpha_\theta\Delta T\right)-\in_{11}\frac{\partial\phi}{\partial x},$$

$$M_x=\frac{h^3}{12}\left(C_{11}\left(-z\frac{\partial^2 w}{\partial x^2}\right)+C_{12}\left(-z\frac{\partial^2 w}{R^2\partial\theta^2}\right)\right),$$

$$M_\theta=\frac{h^3}{12}\left(C_{12}\left(-z\frac{\partial^2 w}{\partial x^2}_x\right)+C_{22}\left(-z\frac{\partial^2 w}{R^2\partial\theta^2}\right)\right),\qquad(24)$$

$$M_{x\theta}=\frac{h^3}{12}C_{66}\left(-2z\frac{\partial^2 w}{R\partial\theta\partial x}\right)$$

The kinetic energy of system may be written as:

$$K=\frac{\rho}{2}\int_v\left((\frac{\partial u}{\partial t})^2+(\frac{\partial v}{\partial t})^2+(\frac{\partial w}{\partial t})^2\right)dV,\qquad(25)$$

The second type of total energy to be verified is the work done by applied force, expressed as (Heydari e al. 2015):

$$W_{vs}=\int(F_e w)dA=\int\left(\left(k_W w-k_g\nabla^2 w\right)\right)wdA,\qquad(26)$$

Applying Hamilton principle and rearranging the governing equation in mechanical displacement directions (u, v and w) as well as electric potential (∅), yield the following four coupled electro-mechanical equations:

$$(hC_{11})\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2}\right)+\frac{hC_{12}}{R}\left(\frac{\partial^2 v}{\partial x\partial\theta}+\frac{\partial w}{\partial x}+\frac{\partial w}{R\partial\theta}\frac{\partial^2 w}{\partial x\partial\theta}\right)$$
$$+\frac{hC_{66}}{R}\left(\frac{\partial^2 u}{R\,\partial\theta^2}+\frac{\partial^2 v}{\partial x\partial\theta}+\frac{\partial w}{R\,\partial x}\frac{\partial^2 w}{\partial\theta^2}+\frac{\partial w}{\partial x}+\frac{\partial w}{R\,\partial\theta}\frac{\partial^2 w}{\partial x\partial\theta}\right)+he_{11}\frac{\partial^2\phi}{\partial x^2}=0\qquad(27)$$

$$\frac{hC_{12}}{R}\left(\frac{\partial^2 u}{\partial x\partial\theta}+\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x\partial\theta}\right)+\frac{hC_{22}}{R^2}\left(\frac{\partial^2 v}{\partial\theta^2}+\frac{\partial w}{\partial\theta}+\frac{\partial w}{R\,\partial\theta}\frac{\partial^2 w}{\partial\theta^2}\right)$$
$$+hC_{66}\left(\frac{\partial^2 u}{R\,\partial x\partial\theta}+\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 w}{R\,\partial\theta\,\partial x}\frac{\partial w}{\partial x}+\frac{\partial w}{R\,\partial\theta}\frac{\partial^2 w}{\partial x^2}\right)+\frac{he_{12}}{R}\frac{\partial^2\phi}{\partial x\partial\theta}=0,\qquad(28)$$

$$\frac{h^3}{12}\left(-C_{11}\frac{\partial^4 w}{\partial x^4}-\frac{C_{12}}{R^2}\frac{\partial^4 w}{\partial x^2\partial\theta^2}\right)+\frac{h^3C_{66}}{3R^2}\left(-\frac{\partial^4 w}{\partial x^2\partial\theta^2}\right)+\frac{h^3}{12R^2}\left(-C_{12}\frac{\partial^4 w}{\partial x^2\partial\theta^2}-\frac{C_{66}}{R^2}\frac{\partial^4 w}{\partial\theta^4}\right)$$
$$-\left(\frac{hC_{12}}{R}\right)\left(\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right)-\frac{hC_{22}}{R}\left(\frac{\partial v}{R\partial\theta}+\frac{w}{R}+\frac{1}{2R^2}\left(\frac{\partial w}{\partial\theta}\right)^2\right)$$
$$-\left(\frac{hC_{12}\alpha_x}{R^2}+\frac{hC_{22}\alpha_\theta}{R^2}\right)\frac{\partial^2 w}{\partial\theta^2}\Delta T-\left(N_x^M+hC_{11}\alpha_x+hC_{12}\alpha_\theta\right)\frac{\partial^2 w}{\partial x^2}\Delta T\qquad(29)$$
$$+\begin{pmatrix}-k_0\left(1-\beta\exp\left(-\alpha x^2\right)\right)w+k_\xi\left(\cos^2\theta w_{,xx}+2\cos\theta\sin\theta w_{,yx}+\sin^2\theta w_{,yy}\right)\\+k_\eta\left(\sin^2\theta w_{,xx}-2\sin\theta\cos\theta w_{,yx}+\cos^2\theta w_{,yy}\right)\end{pmatrix}-\frac{he_{12}}{R}\frac{\partial\phi}{\partial x}=0,$$

$$-\frac{\partial^2\phi}{\partial x^2}+\left(\frac{e_{11}}{\in_{11}}\right)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2}\right)+\frac{e_{12}}{R\in_{11}}\left(\frac{\partial^2 v}{\partial x\partial\theta}+\frac{\partial w}{\partial x}+\frac{\partial w}{R\partial\theta}\frac{\partial^2 w}{\partial x\partial\theta}\right)=0\qquad(30)$$

3. DQM

There is a lot of numerical method to solve the initial-and/or boundary value problems which occur in engineering domain. Some of the common numerical methods are FEM, Galerkin method, finite difference method (FDM), DQM, finite element method (FEM) and finite difference method (FDM) for higher-order modes require to a great number of grid points. Therefore these solution methods for all these points need to more CPU time, while the DQM has several benefits that are listed as below (Heydari et al. 2015, Arani et al. 2015).

1. DQM is a powerful method which can be used to solve numerical problems in the analysis of structural and dynamical systems.
2. The accuracy and convergence of the DQM is higher than FEM.
3. DQM is an accurate method for solution of nonlinear differential equations in approximation of the derivatives.
4. This method can easily and exactly satisfy a variety of boundary conditions and require much less formulation and programming effort.
5. Recently, DQM has been extended to handle irregular shaped.

Due to the above striking merits of the DQM, in recent years the method has become increasingly popular in the numerical solution of problems in engineering and physical science. Hence, DQM is employed which in essence approximates the partial derivative of a function, with respect to a spatial variable at a given discrete point, as a weighted linear sum of the function values at all discrete points chosen in the solution domain of the spatial variable. Let F be a function representing u, v, w and $\emptyset$ with respect to variables x and Θ in the following domain of (0<x<L, 0< Θ<2π) having $N_x \times N_\Theta$ grid points along these variables. The n[th]-order partial derivative of F(x, Θ) with respect to x, the m[th]-order partial derivative of F(x, Θ) with respect to Θ and the (n + m)[th]-order partial derivative of F(x, Θ) with respect to both x and Θ may be expressed discretely at the point (x_i, Θ_i) as:

$$\frac{d^n f_x(x_i,\theta_j)}{dx^n} = \sum_{k=1}^{N_x} A_{ik}^{(n)} f(x_k,\theta_j) \qquad n = 1,\ldots,N_x - 1. \tag{31}$$

$$\frac{d^m f_y(x_i,\theta_j)}{d\theta^m} = \sum_{l=1}^{N_\theta} B_{jl}^{(m)} f(x_i,\theta_l) \qquad m = 1,\ldots,N_\theta - 1. \tag{32}$$

$$\frac{d^{n+m} f_{xy}(x_i,\theta_j)}{dx^n d\theta^m} = \sum_{k=1}^{N_x}\sum_{l=1}^{N_\theta} A_{ik}^{(n)} B_{jl}^{(m)} f(x_k,\theta_l). \tag{33}$$

A more superior choice for the positions of the grid points is Chebyshev polynomials as expressed:

$$x_i = \frac{L}{2}\left[1-\cos\left(\frac{i-1}{N_x-1}\right)\pi\right] \qquad i = 1,\ldots,N_x \tag{34}$$

$$\theta_i = \frac{2\pi}{2}\left[1-\cos\left(\frac{i-1}{N_\theta-1}\right)\pi\right] \qquad i = 1,\ldots,N_\theta \tag{35}$$

Also $A_{ik}^{(n)}$ and $B_{jl}^{(m)}$ are the weighting coefficients associated with n[th]-order partial derivative of F(x, Θ) with respect to x at the discrete point x_i and m[th]-order derivative with respect to Θ at Θ_i respectively which may be calculated as:

$$A_{ij}^{(1)} = \begin{cases} \dfrac{M(x_i)}{(x_i-x_j)M(x_j)} & for \quad i \neq j, \quad i,j = 1,2,\ldots,N_x \\ -\sum\limits_{\substack{j=1 \\ i\neq j}}^{N_x} A_{ij}^{(1)} & for \quad i = j, \quad i,j = 1,2,\ldots,N_x \end{cases} \tag{36}$$

$$B_{ij}^{(1)} = \begin{cases} \dfrac{P(\theta_i)}{(\theta_i-\theta_j)P(\theta_j)} & for \quad i \neq j, \quad i,j = 1,2,\ldots,N_\theta, \\ -\sum\limits_{\substack{j=1 \\ i\neq j}}^{N_\theta} B_{ij}^{(1)} & for \quad i = j, \quad i,j = 1,2,\ldots,N_\theta \end{cases} \tag{37}$$

Where

$$M(x_i) = \prod_{\substack{j=1 \\ j\neq i}}^{N_x} (x_i - x_j) \tag{38}$$

$$P(\theta_i) = \prod_{\substack{j=1 \\ j\neq i}}^{N_\theta} (\theta_i - \theta_j) \tag{39}$$

For higher order derivatives we have:

$$A_{ij}^{(n)} = n\left(A_{ii}^{(n-1)} A_{ij}^{(1)} - \frac{A_{ij}^{(n-1)}}{(x_i - x_j)}\right) \tag{40}$$

$$B_{ij}^{(m)} = m\left(B_{ii}^{(m-1)}B_{ij}^{(1)} - \frac{B_{ij}^{(m-1)}}{(\theta_i - \theta_j)} \right) \tag{41}$$

However, applying below dimensionless parameters:

$$\begin{aligned}
&\gamma = \frac{h}{L},\ \ \xi = \frac{x}{L},\quad \beta = \frac{h}{R}, \{\bar{u},\bar{v},\bar{w}\} = \frac{\{u,v,w\}}{h} \\
&\bar{C}_{ij} = \frac{C_{ij}}{C_{11}},\ \ K_w = \frac{hk_w}{C_{11}},\quad K_g = \frac{k_g}{hC_{11}}, \\
&\Phi = \frac{\phi}{\Phi_0},\quad \Phi_0 = h\sqrt{\frac{C_{11}}{\in_{11}}},\quad \bar{e}_{ij} = \frac{e_{ij}}{\sqrt{C_{11}\in_{11}}},\ \bar{t} = \frac{t}{h\sqrt{\frac{\rho_f}{C_{s11}}}},
\end{aligned} \tag{42}$$

And DQM, the governing equations may be written as:

$$\begin{aligned}
&(\gamma^2)\left(\sum_{k=1}^{N_x} A^{(2)}{}_{ik}\bar{u}(x_k,\theta_j) + \sum_{k=1}^{N_x} A^{(1)}{}_{ik}\bar{w}(x_k,\theta_j)\sum_{k=1}^{N_x} A^{(2)}{}_{ik}\bar{w}(x_k,\theta_j)\right) + \beta\gamma\bar{C}_{12}\left(\sum_{k=1}^{N_x}\sum_{p=1}^{N_\theta} A^{(1)}{}_{ik}B^{(1)}{}_{jp}\bar{w}(x_k,\theta_j) + \sum_{k=1}^{N_x} A^{(1)}{}_{ik}\bar{w}(x_k,\theta_j)\right. \\
&\left.\beta\sum_{p=1}^{N_\theta} B^{(1)}{}_{jp}\bar{w}(x_k,\theta_j)\sum_{k=1}^{N_x}\sum_{p=1}^{N_\theta} A^{(1)}{}_{ik}B^{(1)}{}_{jp}\bar{w}(x_k,\theta_j)\right) + \beta\bar{C}_{66}\left(\beta\sum_{p=1}^{N_\theta} B^{(2)}{}_{jp}\bar{u}(x_k,\theta_j) + \gamma\sum_{k=1}^{N_x}\sum_{p=1}^{N_\theta} A^{(1)}{}_{ik}B^{(1)}{}_{jp}\bar{v}(x_k,\theta_j)\right. \\
&\left.+\beta\sum_{k=1}^{N_x} A^{(1)}{}_{ik}\bar{w}(x_k,\theta_j)\sum_{p=1}^{N_\theta} B^{(2)}{}_{jp}\bar{w}(x_k,\theta_j) + \gamma\sum_{k=1}^{N_x} A^{(1)}{}_{ik}\bar{w}(x_k,\theta_j) + \beta\sum_{p=1}^{N_\theta} B^{(1)}{}_{jp}\bar{w}(x_k,\theta_j)\sum_{k=1}^{N_x}\sum_{p=1}^{N_\theta} A^{(1)}{}_{ik}B^{(1)}{}_{jp}\bar{v}(x_k,\theta_j)\right) \\
&+\gamma^2\bar{e}_{11}\sum_{k=1}^{N_x} A^{(2)}{}_{ik}\Phi(x_k,\theta_j) = 0,
\end{aligned} \tag{43}$$

$$\begin{aligned}
&\beta\bar{C}_{12}\left(\gamma\sum_{k=1}^{N_x}\sum_{p=1}^{N_\theta} A^{(1)}{}_{ik}B^{(1)}{}_{jp}\bar{u}(x_k,\theta_j) + \gamma^2\sum_{k=1}^{N_x} A^{(1)}{}_{ik}\bar{w}(x_k,\theta_j)\sum_{k=1}^{N_x}\sum_{p=1}^{N_\theta} A^{(1)}{}_{ik}B^{(1)}{}_{jp}\bar{w}(x_k,\theta_j)\right) + \beta^2\bar{C}_{22}\left(\sum_{p=1}^{N_\theta} B^{(2)}{}_{jp}\bar{v}(x_k,\theta_j)\right. \\
&\left.+\sum_{p=1}^{N_\theta} B^{(1)}{}_{jp}\bar{w}(x_k,\theta_j) + \beta\sum_{p=1}^{N_\theta} B^{(1)}{}_{jp}\bar{w}(x_k,\theta_j)\sum_{p=1}^{N_\theta} B^{(2)}{}_{jp}\bar{w}(x_k,\theta_j)\right) + \gamma\bar{C}_{66}\left(\beta\sum_{k=1}^{N_x}\sum_{p=1}^{N_\theta} A^{(1)}{}_{ik}B^{(1)}{}_{jp}\bar{u}(x_k,\theta_j) + \gamma\sum_{k=1}^{N_x} A^{(2)}{}_{ik}\bar{v}(x_k,\theta_j)\right. \\
&\left.+\beta\gamma\sum_{k=1}^{N_x}\sum_{p=1}^{N_\theta} A^{(1)}{}_{ik}B^{(1)}{}_{jp}\bar{w}(x_k,\theta_j)\sum_{k=1}^{N_x} A^{(1)}{}_{ik}\bar{w}(x_k,\theta_j) + \beta\gamma\sum_{p=1}^{N_\theta} B^{(1)}{}_{jp}\bar{w}(x_k,\theta_j)\sum_{k=1}^{N_x} A^{(2)}{}_{ik}\bar{w}(x_k,\theta_j)\right) \\
&+\gamma_s\bar{C}_{s66}\left(\beta_s\sum_{k=1}^{N_x}\sum_{p=1}^{N_\theta} A^{(1)}{}_{ik}B^{(1)}{}_{jp}\bar{u}(x_k,\theta_j) + \sum_{k=1}^{N_x} A^{(2)}{}_{ik}\bar{v}(x_k,\theta_j) + \beta\bar{e}_{12}\sum_{k=1}^{N_x}\sum_{p=1}^{N_\theta} A^{(1)}{}_{ik}B^{(1)}{}_{jp}\Phi(x_k,\theta_j) = 0,\right.
\end{aligned} \tag{44}$$

$$\begin{aligned}
&\frac{\gamma^2}{12}\left(-\gamma^2\sum_{k=1}^{N_x} A^{(4)}{}_{ik}\bar{w}(x_k,\theta_j) - \bar{C}_{12}\beta^2\sum_{k=1}^{N_x}\sum_{p=1}^{N_\theta} A^{(2)}{}_{ik}B^{(2)}{}_{jp}\bar{w}(x_k,\theta_j)\right) - \frac{\gamma^2\beta^2\bar{C}_{66}}{3}\left(\sum_{k=1}^{N_x}\sum_{p=1}^{N_\theta} A^{(2)}{}_{ik}B^{(2)}{}_{jp}\bar{w}(x_k,\theta_j)\right) \\
&+\frac{1}{12}\left(-\beta^4\bar{C}_{66}\sum_{k=1}^{N_x}\sum_{p=1}^{N_\theta} A^{(2)}{}_{ik}B^{(2)}{}_{jp}\bar{w}(x_k,\theta_j) - \gamma^2\beta^2\bar{C}_{12}\sum_{p=1}^{N_\theta} B^{(4)}{}_{jp}\bar{w}(x_k,\theta_j)\right) - \gamma\beta\bar{C}_{12}\left(\sum_{k=1}^{N_x} A^{(1)}{}_{ik}\bar{u}(x_k,\theta_j) + \frac{\gamma}{2}\sum_{k=1}^{N_x} A^{(1)}{}_{ik}\bar{w}(x_k,\theta_j)\sum_{k=1}^{N_x} A^{(1)}{}_{ik}\bar{w}(x_k,\theta_j)\right) \\
&-\beta\bar{C}_{22}\left(\beta\sum_{p=1}^{N_\theta} B^{(1)}{}_{jp}\bar{v}(x_k,\theta_j) + \beta\,\bar{w}(x_k,\theta_j) + \frac{\beta^2}{2}\sum_{p=1}^{N_\theta} B^{(1)}{}_{jp}\bar{w}(x_k,\theta_j)\sum_{p=1}^{N_\theta} B^{(1)}{}_{jp}\bar{w}(x_k,\theta_j)\right) - \left(\left(\beta^2\alpha_x\bar{C}_{12} + \beta^2\bar{C}_{22}\alpha_\theta\right)\Delta T\sum_{p=1}^{N_\theta} B^{(2)}{}_{jp}\bar{w}(x_k,\theta_j)\right. \\
&\left.-\left(\gamma^2\alpha_x + \gamma^2\bar{C}_{12}\alpha_\theta\right)\sum_{k=1}^{N_x} A^{(2)}{}_{ik}\bar{w}(x_k,\theta_j)\right)\Delta T - k_0\left(1-\beta\exp\left(-\alpha x_{ik}{}^2\right)\right)\bar{w}(x_k,\theta_j) \\
&+k_\xi\left(\cos^2\theta\sum_{k=1}^{N_x} A^{(2)}{}_{ik}\bar{w}(x_k,\theta_j) + 2\cos\theta\sin\theta\left(\sum_{k=1}^{N_x}\sum_{p=1}^{N_\theta} A^{(2)}{}_{ik}B^{(2)}{}_{jp}\bar{w}(x_k,\theta_p)\right) + \sin^2\theta\sum_{p=1}^{N_\theta} B^{(2)}{}_{jp}\bar{w}(x_p,\theta_j)\right) \\
&+k_\eta\left(\sin^2\theta\sum_{k=1}^{N_x} A^{(2)}{}_{ik}\bar{w}(x_k,\theta_j) - 2\sin\theta\cos\theta\left(\sum_{k=1}^{N_x}\sum_{p=1}^{N_\theta} A^{(2)}{}_{ik}B^{(2)}{}_{jp}\bar{w}(x_k,\theta_p)\right) + \cos^2\theta\sum_{p=1}^{N_\theta} B^{(2)}{}_{jp}\bar{w}(x_p,\theta_j)\right) \\
&+\gamma\beta\bar{e}_{12}\sum_{k=1}^{N_x} A^{(1)}{}_{ik}\Phi(x_k,\theta_j) = \ddot{\bar{w}}(x_i,\theta_j),
\end{aligned} \tag{45}$$

$$-\sum_{k=1}^{N_x} A^{(2)}{}_{ik}\Phi(x_k,\theta_j)+\bar{e}_{11}\left(\sum_{k=1}^{N_x} A^{(2)}{}_{ik}\bar{u}(x_k,\theta_j)+\gamma\sum_{k=1}^{N_x} A^{(1)}{}_{ik}\bar{w}(x_k,\theta_j)\sum_{k=1}^{N_x} A^{(2)}{}_{ik}\bar{w}(x_k,\theta_j)\right)$$

$$+\frac{\beta\,\bar{e}_{12}}{\gamma}\left(\sum_{k=1}^{N_x}\sum_{p=1}^{N_\theta} A^{(1)}{}_{ik}B^{(1)}{}_{jp}\bar{v}(x_k,\theta_j)+\sum_{k=1}^{N_x} A^{(1)}{}_{ik}\bar{w}(x_k,\theta_j)+\frac{\beta}{\gamma}\sum_{p=1}^{N_\theta} B^{(1)}{}_{jp}\bar{w}(x_k,\theta_j)\sum_{k=1}^{N_x}\sum_{p=1}^{N_\theta} A^{(1)}{}_{ik}B^{(1)}{}_{jp}\bar{v}(x_k,\theta_j)\right)=0 \tag{46}$$

According to HDQM, mechanical and electrical boundary conditions may be written as:

$$\begin{cases} w_{i1}=v_{i1}=u_{i1}=0, & \sum_{j=1}^{N_\theta} A_{2j}w_{ji}=0 \\ w_{N_x i}=v_{N_x i}=u_{N_x i}=0, & \sum_{j=1}^{N_\theta} A_{(N_x-1)j}w_{ji}=0 \end{cases} \quad for \quad i=1\ldots N_\theta \tag{47}$$

Applying these boundary conditions into the governing yields the following coupled assembled matrix equations:

$$\left(\left[\underbrace{K_L+K_{NL}}_{K}\right]+\Omega^2[M]\right)\begin{Bmatrix}\{d_b\}\\ \{d_d\}\end{Bmatrix}=0, \tag{48}$$

Where K_L, K_{NL} and M are linear stiffness matrix, nonlinear stiffness matrix and mass matrix, respectively. Also, d_b and d_d represent boundary and domain points expressed as:

$$\{d_b\}=\{\bar{u}_{i1},\bar{v}_{i1},\bar{w}_{i1},\bar{w}_{i2},\Phi_{i1},\bar{u}_{iN_\theta},\bar{v}_{iN_\theta},\bar{w}_{iN_\theta},\bar{w}_{i(N_\theta-1)},\Phi_{iN_\theta}\} \quad i=1,\ldots,N_x$$
$$\{d_d\}=\{\bar{u}_{ij},\bar{v}_{ij},\bar{w}_{i(j+1)},\Phi_{ij}\} \quad i=1,\ldots,N_x,\; j=2,\ldots,N_x-1 \tag{49}$$

Finally, based on an iterative method and eigenvalue problem, the frequency of structure may be obtained.

4. Numerical result

In order to obtain the frequency for considered pipe embedded in the Pasternak foundation, DQM was used in conjunction with a program being written in MATLAB, where the effect of volume percent of DWBNNTs, orientation angle of DWBNNTs and elastic medium were investigated. Mechanical and electrical characteristics of PVDF matrix and DWBNNTs reinforce are assumes as (Mosallaie Barzoki et al. 2013):

$$C^m_{11}=10.64\,GPa \quad C^m_{23}=3.98\,GPa \quad E_p=1.8\,TPa \quad \nu_p=0.34 \quad C^m_{22}=23.6\,GPa$$
$$C^m_{44}=6.43\,GPa \quad C^m_{12}=1.92\,GPa \quad e^p_{31}=0.95\,C/m^2 \quad C^m_{13}=2.19\,GPa$$
$$e^m_{31}=-0.13\,C/m^2 \; e^m_{32}=-0.145\,C/m^2 \; e^m_{31}=-0.135\,C/m^2$$

The effect of orientation angle of DWBNNTs in pipe on the nonlinear frequency of pipe versus thickness to radius of pipe is showed in Fig. 1 that can be seen, with increasing the orientation angle of DWBNNTs, the nonlinear frequency decreases. Hence, maximum and minimum frequency are related to $\theta=0$ and $\theta=\pi/2$ respectively. It is due to the fact that in $\theta=0$, the polarization of pipe and DWBNNTs are in a one direction and consequently, the stiffness of structure is maximum.

The effect of volume percent of DWBNNTs in pipe on the nonlinear frequency versus aspect ratio of pipe and orientation angle of DWBNNTs is illustrated in Figs. 2 and 3, respectively. It can be found that with increasing the volume percent of DWBNNTs, the nonlinear frequency increases. It is due to the fact that with increasing volume percent of DWBNNTs in pipe, the stiffness of structure increases. Hence, the DWBNNT volume fraction and orientation angle in pipe are effective controlling parameters for vibration of the pipes.

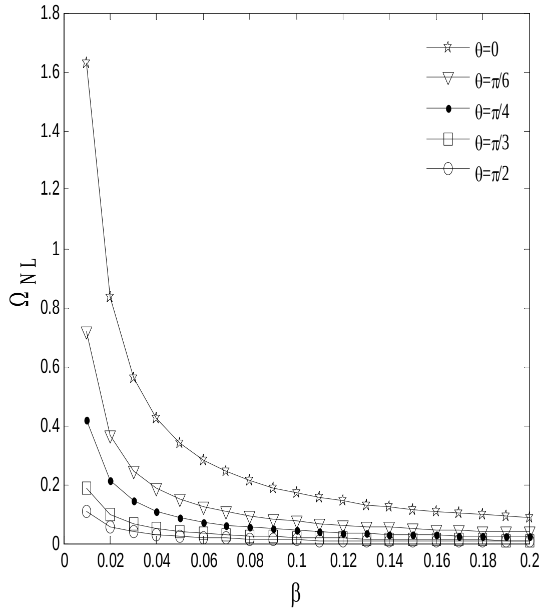

Fig. 1: The effect of orientation angle of DWBNNTs on the nonlinear frequency (Against aspect ratio)

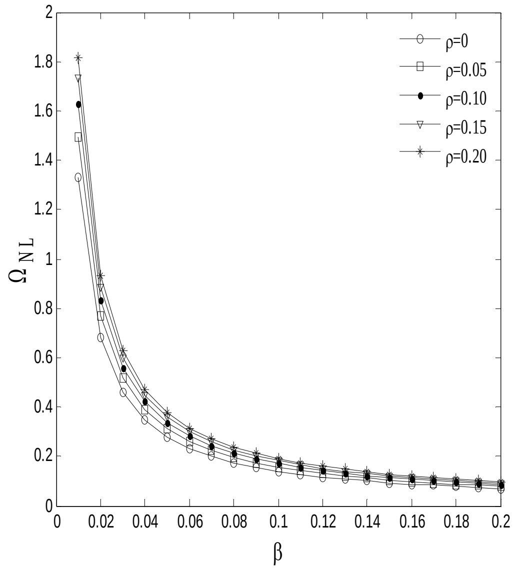

Fig. 2: The effect of volume percent of DWBNNTs on the nonlinear frequency (Against aspect ratio)

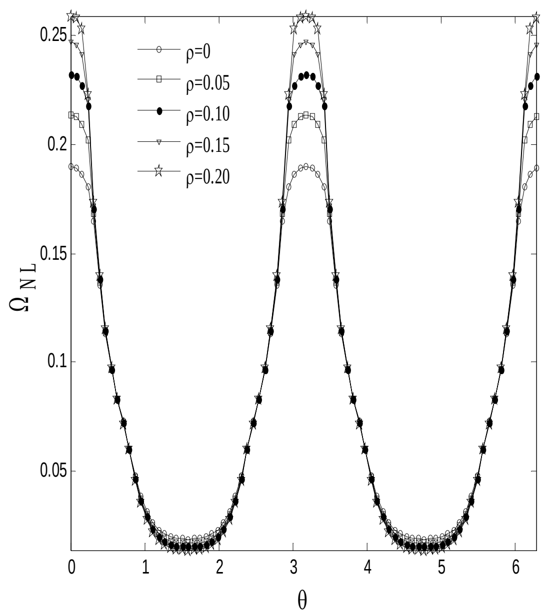

Fig. 3: The effect of volume percent of DWBNNTs on the nonlinear frequency (Against orientation angle of DWBNNTs)

Figs. 4 and 5 illustrate the influence of elastic medium, including Winkler and Pasternak modules, on the frequency, along the aspect ratio of pipe and orientation angle of DWBNNTs. Obviously, the elastic medium type has a significant effect on vibration of the pipe, since the frequency of the system in the case of without elastic medium are lower than other cases. It can be concluded that the frequency for Pasternak model is higher than Winkler one. The above results are reasonable, since the Pasternak medium considers not only the normal stresses (i.e. Winkler foundation) but also the transverse shear deformation and continuity among the spring elements.

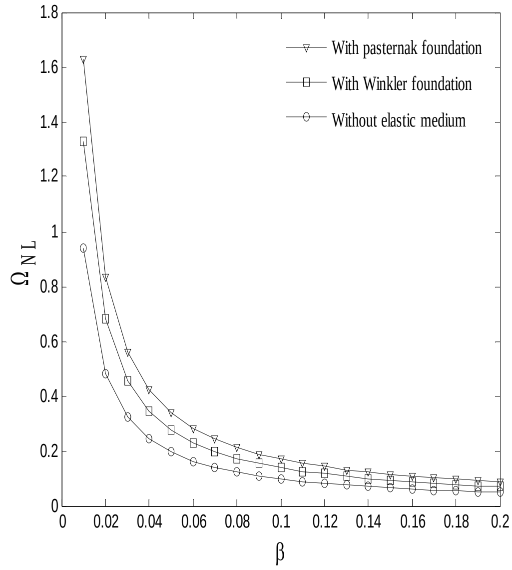

Fig. 4: The effect of elastic foundation on the nonlinear (Against aspect ratio)

5. Conclusion

Nonlinear vibration of embedded piezoelectric pipes reinforced with DWBNNTs is the main contribution of present work. The pipe and elastic medium are simulated with cylindrical shell theory and Pasternak model, respectively.

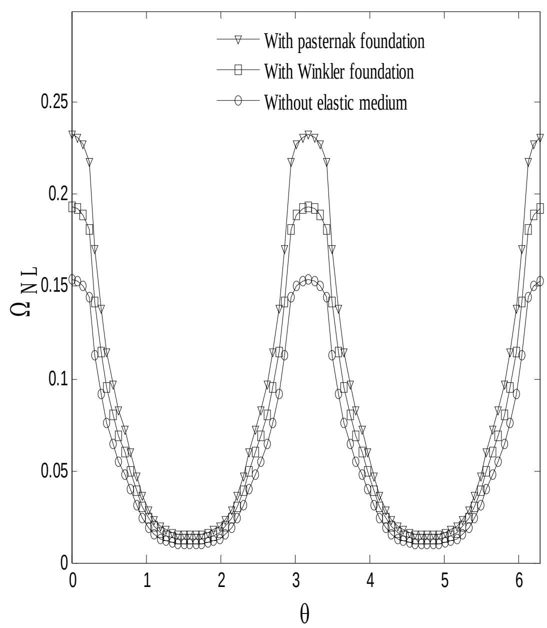

Fig. 5: The effect of elastic foundation on the nonlinear frequency (Against orientation angle of DWBNNTs)

The nonlinear frequency is calculated with DQM and the effects of different parameters such as volume percent and orient DWBNNTs, geometrical parameters and elastic medium are discussed. The following results are the main concussions of this work:

- With increasing volume percent of DWBNNTs in pipe, the frequency increases.

- With increasing the orientation angle of DWBNNTs from 0 to $\pi/2$, the nonlinear frequency decreases.
- The maximum and minimum frequency are related to $\theta = 0$ and $\theta = \pi/2$, respectively.
- The frequency for Pasternak model is higher than Winkler one.
- It is hoped that the obtained results might be useful for the design and improvement of smart devices applying nanotechnology.

Acknowledgments

The authors are grateful to the Islamic Azad University Branch of Khomein for supporting this work. They would like also to thank the Iranian Nanotechnology Development Committee for their financial support.

References

Amabili M and Garziera R (2002). Vibrations of circular cylindrical shells with non-uniform constraints, elastic bed and added mass. Part II: shells containing or immersed in axial flow. Journal of Fluids and Structures, 16(1): 31-51.

Barzoki AM Arani AG, Kolahchi R., Mozdianfard MR. and Loghman A (2013). Nonlinear buckling response of embedded piezoelectric cylindrical shell reinforced with BNNT under electro–thermo-mechanical loadings using *HDQM.* Composites Part B: Engineering, 44(1): 722-727.

Bent AA, Hagood NW and Rodgers JP (1995). Anisotropic actuation with piezoelectric fiber composites. Journal of Intelligent Material Systems and Structures, 6(3): 338-349.

Brush DO and Almroth BO (1975). Buckling of bars, plates and shells. McGraw-Hill, New York.

Ghorbanpour Arani A, Kolahchi R and Mosallaie Barzoki A (2011). Effect of material in-homogeneity on electro-thermo-mechanical behaviors of functionally graded piezoelectric rotating shaft. Journal of Applied Mathematical Modeling, 35(6): 2771-2789.

Ghorbanpour Arani A, Kolahchi R, Mosallaie Barzoki A and Loghman A (2012). Electro-thermo-mechanical behaviors of FGPM spheres using analytical method and ANSYS software. Applied Mathematical Modeling, 36(1): 139-157.

Ghorbanpour Arani A, Kolahchi R. and Zarei MS (2015). Visco-surface-nonlocal piezoelasticity effects on nonlinear dynamic stability of graphene sheets integrated with ZnO sensors and actuators using refined zigzag theory. Composite Structures, 132 (2015): 506-526.

Heydari MM, Bidgoli AH, Golshani HR, Beygipoor G and Davoodi A (2015). Nonlinear bending analysis of functionally graded CNT-reinforced composite Mindlin polymeric temperature-dependent plate resting on orthotropic elastomeric medium using GDQM. Nonlinear Dynamics, 79(2): 1425-1441.

Kadoli R and Ganesan N (2003). Free vibration and buckling analysis of composite cylindrical shells conveying hot fluid. Composite structures, 60(1): 19-32.

Matsunaga H (2007). Vibration and buckling of cross-ply laminated composite circular cylindrical shells according to a global higher-order theory. International journal of mechanical sciences, 49(9): 1060-1075.

Messina A and Soldatos KP (1999). Vibration of completely free composite plates and cylindrical shell panels by a higher-order theory. International Journal of Mechanical Sciences, 41(8): 891-918.

Mohammadi H, Mahzoon M, Mohammadi M and Mohammadi M (2014). Postbuckling instability of nonlinear nanobeam with geometric imperfection embedded in elastic foundation. Nonlinear Dynamics, 76(4): 2005-2016.

Nguyen-Van H, Mai-Duy N, Karunasena W and Tran-Cong T (2011). Buckling and vibration analysis of laminated composite plate/shell structures via a smoothed quadrilateral flat shell element with in-plane rotations. Computers and Structures, 89(7): 612-625.

Païdoussis MP (1998). Fluid–Structure Interactions: Slender Structures and Axial Flow, Elsevier Academic Press, London.

Païdoussis MP and Denise JP (1972). Flutter of thin cylindrical shells conveying fluid. Journal of Sound and Vibration, 20(1): 9-26.

Rahmani O, Khalili SMR and Malekzadeh K (2010). Free vibration response of composite sandwich cylindrical shell with flexible core. Composite Structures, 92(5): 1269-1281.

Tan P and Tong L (2001). Micro-electro mechanics models for piezoelectric-fiber-reinforced composite materials. Composites science and technology, 61(5): 759-769.

Topolov VY and Bowen CR (2008). Electromechanical properties in composites based on ferroelectrics. Springer Science and Business Media. London.